Theories of Failure

Failure is generally perceived to be fracture or complete separation of a member. However, failure may also occur due to excessive deformation (elastic or inelastic) or a variety of other reasons.

During the latter part of the 19th century and continuing up to the present, a number of basic failure

theories were proposed and tested on a few materials. Most of the theories were based on the assumption that failure occurs when some physical variable such as stress, strain, or energy reaches a limiting value.

Stress Theories

Maximum Principal Stress Theory (Rankine, Lamé) (Fig. 1) Applied satisfactorily to many brittle materials, the theory is based on a limiting normal stress. Failure occurs when the normal stress reaches a specified upper limit.[1]

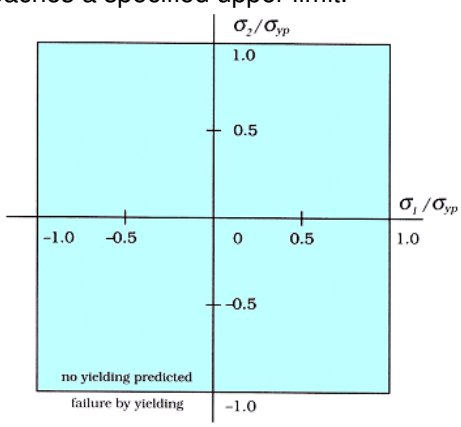


Fig. (1)

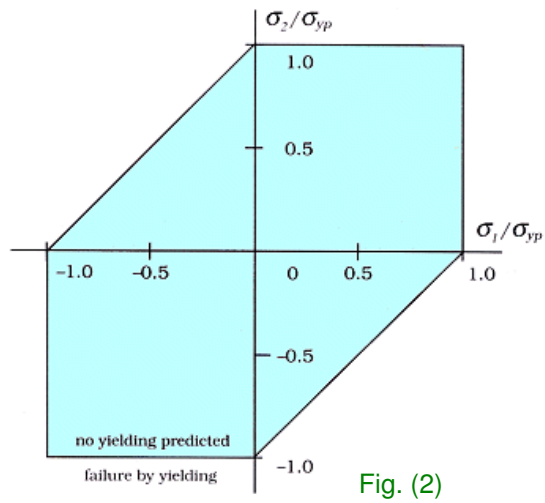


Fig. (2)

Maximum Shear Stress Theory (Tresca, Guest, Coulomb) Fig. (2)

Applied satisfactorily to ductile materials, the theory is based on the concept of limiting shearing stress at which failure occurs.[1]

Failure by yielding in a more complicated loading situation is assumed to occur when the maximum shearing stress in the material reaches a value equal to the maximum shearing stress in a tension test at yield.

$$\tau_{max} = \frac{(\sigma_{max} - \sigma_{min})}{2} = \tau_{yp} = \frac{\sigma_{yp}}{2}$$

This yield criterion gives good agreement with experimental results for ductile materials; because of its simplicity, it is the most often used yield theory.[2] The main objection to this theory is that it ignores the possible effect of the intermediate principal stress, σ_2. However, only one other theory, the maximum distortional strain energy theory, predicts yielding better than does the Tresca theory, and the differences between the two theories are rarely more than 15%.

Maximum Principal Strain Theory (Saint-Venant) (Fig. 3)

The theory is based on the assumption that inelastic behavior or failure is governed by a specified maximum normal strain.[1] Failure will occur at a particular part in a body subjected to an arbitrary state of strain when the normal strain reaches a limiting level.

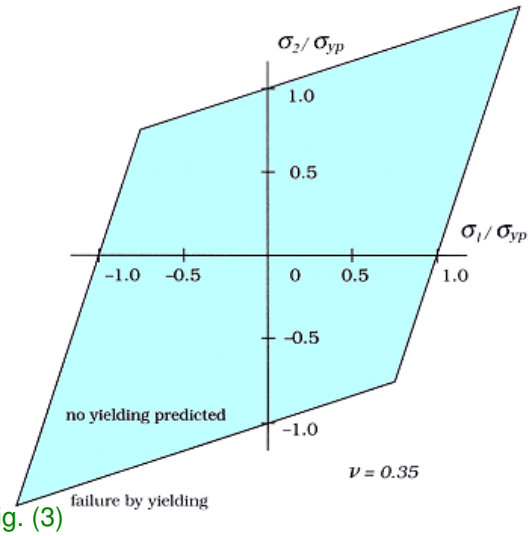


Fig. (3)

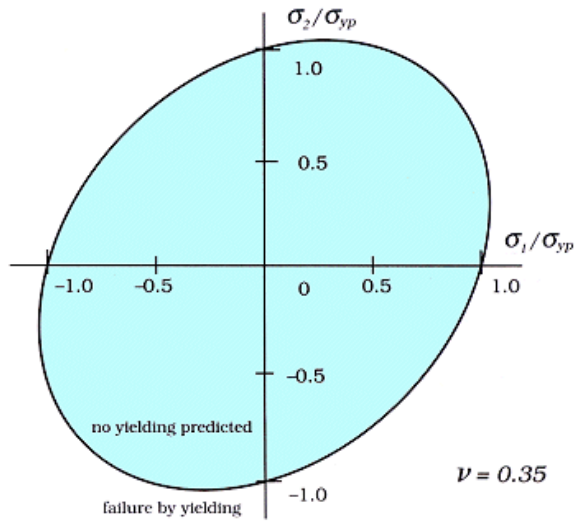


Fig. (4)

Total Strain Energy Theory (Beltrami-Haigh) Fig. (4)

Applicable to many types of materials, the theory predicts failure or inelastic action at a point when the strain energy per unit volume exceeds a specified limit.

Maximum Distortion Energy Theory (Huber-Henky-von Mises) (Fig. 5)

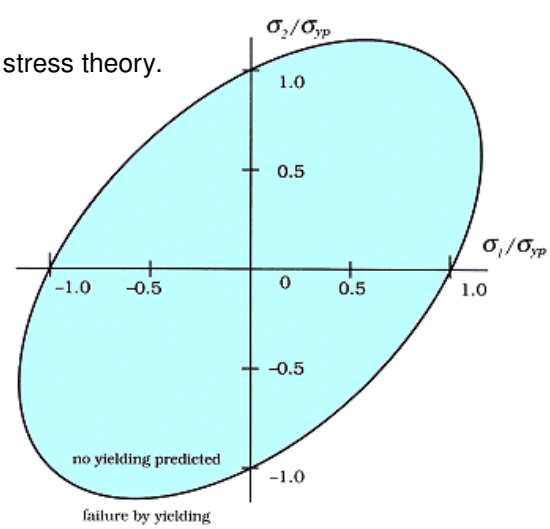
The theory is based on a limiting energy of distortion, i.e. energy associated with shear strains.[1]

Strain energy can be separated into energy associated with volume change and energy associated with distortion of the body. The maximum distortion energy failure theory assumes failure by yielding in a more complicated loading situation to occur when the distortion energy in the material reaches the same value as in a tension test at yield.

This theory provides the best agreement between experiment and theory and, along the Tresca theory, is very widely used today.[2]

Note: This theory gives the same results as the octahedral shear stress theory.

Fig. (5)



Summary

Of the failure criteria, the Tresca is the most conservative for all materials, the von Mises the most representative for ductile materials, and the Rankine the best fit for brittle materials.