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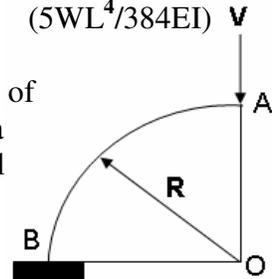
**Mechanics of Materials-II**

6<sup>th</sup> Semester---- (Session 2007)

**Problem Sheet #3**

**Pb.1** Find the deflection of a simply-supported beam of length  $L$  and flexural rigidity  $EI$  carrying a load  $W$  uniformly distributed over the whole beam using virtual work principle or Castigliano's theorem. (5WL<sup>4</sup>/384EI)

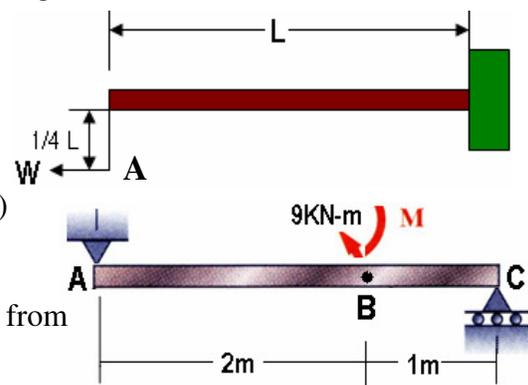
**Pb. 2** A structure is in the form of one quadrant of the thin circular ring of radius 'R' with one end is clamped while other end is loaded by a vertical load as shown in Fig. Determine the horizontal & vertical displacement under the application of vertical load.  
( $\delta_v = \pi VR^3/4EI$ ;  $\delta_H = VR^3/2EI$ )



**Pb.3** For a cantilever beam of length  $L$  and flexural rigidity  $EI$  with load  
(a) uniformly distributed over the whole beam, (b) concentrated at mid span  
Using Castigliano's theorem or virtual work principle determine:  
(i) slope at the free end (WL<sup>2</sup>/6EI, WL<sup>2</sup>/8EI)  
(ii) deflection at the free end (WL<sup>3</sup>/8EI, 5WL<sup>3</sup>/48EI)  
(iii) deflection at the mid-span (17WL<sup>3</sup>/384EI, WL<sup>3</sup>/24EI)

**Pb.4** A simply supported beam of length 'L' and flexural rigidity  $EI$  carries a concentrated load 'w' at a distance of 'd' from the left hand support. Determine the deflection of the beam underneath the load using Castigliano's theorem.  
(wd<sup>2</sup>(L-d)<sup>2</sup>/3EIL)

**Pb.5** Using Castigliano's theorem or virtual work principle determine the horizontal and vertical deflection of point A of the member shown in Fig.  
(13WL<sup>3</sup>/192EI, WL<sup>3</sup>/8EI)



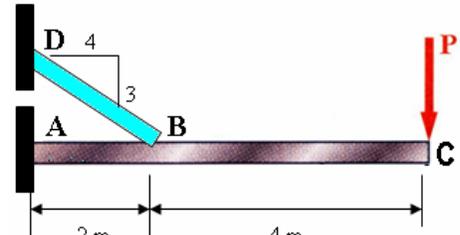
**Pb. 6** A beam of 3 m length is simply-supported at each end and is subjected to a couple of 9KN-m at a point B, 2 m from the left end as shown in Fig. Determine the slope at B.  
Take  $EI=30KN\cdot m^2$ . (0.1 rad)

**Pb. 7** Two identical rectangular section steel members are loaded in the following way:  
(a) one is subjected to an axial tensile force  $W$ , (b) the other is supported as a cantilever and subjected to a load  $W$  at its free end. If each member is required to absorb the same strain energy, compare the maximum stress which this would cause in each case. ( $\sigma_b = 3\sigma_t$ )

**Pb. 8** A hollow shaft, subjected to a pure torque, attains a maximum shear stress of  $\tau$ . Given that the strain energy per volume is  $\tau^2/3G$ , calculate the ratio of shaft diameters. Determine the actual diameter of such a shaft to transmit 4 MW at 110 rpm when the energy stored per unit volume is  $20,000\text{Nm/m}^3$ .  $G=80,000\text{N/mm}^2$ .  
(1.732, 306 mm, 177 mm)

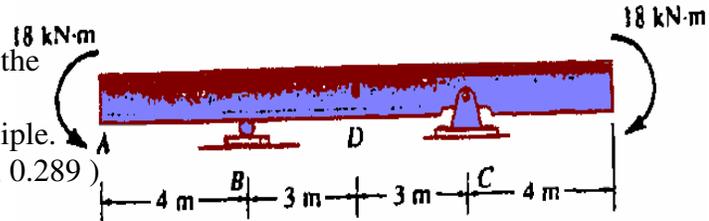
**Pb. 9** Using Castigliano's theorem or virtual work principle find the deflection and rotation at the middle of a cantilever beam of length  $L$  and flexural stiffness  $EI$  carrying a uniformly varying load from zero at the free end to  $W_0$  at the fixed end.  
( $49W_0L^4/3840EI$ ,  $15W_0L^3/384EI$ )

**Pb.10** An aluminium beam is supported by a pin at one end at point A and an inclined aluminium bar at end point D, as shown in Fig. Using Castigliano's theorem find the deflection at C caused by the application of the downward force P of 2 kN at that point. The cross section of the beam is  $0.5\text{cm}^2$ , and that of the bar,  $0.05\text{cm}^2$ .  $E=70\text{GPa}$  and  $I=6000\text{cm}^4$  (for the beam). Neglect deflection caused by shear.  
(19 mm)



**Pb. 11** A beam, fixed at its ends, span 3m, carries a load of 60 kN at 1m from one end. Using Castigliano's theorem find the fixing moments at the ends & the deflection of the load.  $E = 205 \text{ KN/mm}^2$  &  $I = 8000 \text{ cm}^4$  (26.67 kN, 13.33 kNm, 0.362 mm)

**Pb.12** For the steel beam shown in Fig. determine the displacement at 'D' & slope at 'A' using Castigliano's theorem or Virtual work principle.  $I = 125 * 10^6 \text{ mm}^4$  &  $E = 200 \text{ GPa}$  (3.24 mm, 0.289)



**Pb. 13** Determine the horizontal & vertical displacements of point 'C'. There is fixed support at 'A' & EI is constant. ( $4000 / EI \text{ Nm}^3$ ,  $76800 / EI \text{ Nm}^3$ )

