



DEPARTMENT OF MECHANICAL ENGINEERING, University of Engineering & Technology, Lahore (KSK- Campus)

Lab Manual

Mechanics of Materials-II

EXPERIMENT NO. 1

Objective:

To measure the stiffness of a Compression spring and compare it with the theoretical values.

Apparatus:

Compression of spring apparatus, Hangers and Weights,

Summary of Theory:

- Springs
- Types of Compression springs
- Derivation of formula (Castigliano's theorem)



Procedure:

Measure the diameter of wire and outer dia of spring with the help of vernier caliper. Fit the compression spring in the spring support. To fit compression spring, remove the load hanger base by unscrewing the grip knob and base from the rod thread. Loosen or remove the grip knob on the marker and pull the load hanger down until the top can be swung out from the slop by the 50 mm scale. Withdraw the rod upward, insert the new spring and reverse the above procedure to return the apparatus to full working condition.

Load the spring by 5N increments recording the change in length of the spring up to the greatest readable deflection or the max load of 55N. Record the spring dimensions Repeat the same process for other springs and record the readings.

Load-Extension Curve:

Observations & Calculations:

		<u>Spring Data</u>
Wire diameter	=	_____ (mm)
Spring O/D	=	_____ (mm)
Spring Length	=	_____ (mm)
Number of active turns	=	_____
Modulus of rigidity	=	_____ (N/ mm ²)

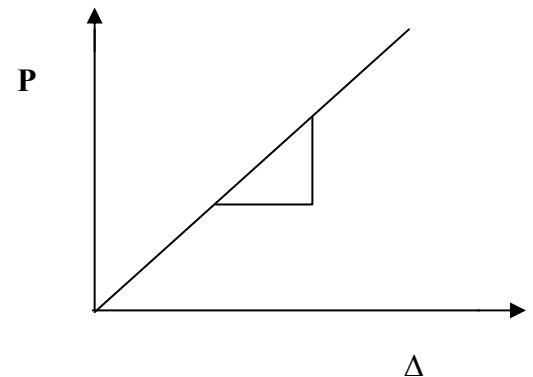


Figure (1)



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$$\text{Stiffness} = W / \Delta = d^4 G / 8N D^3$$

Where d = Wire diameter

N = Number of active turns

D = mean diameter of spring coil (O/D – d)

G = Modulus of rigidity (77 KN/ mm² for spring steels)

No. of Obs.	Load (W) N	Deflection (Δ) mm			Slope from Graph (N/mm)	Theoretical Value = $d^4 G / 8N D^3$ (N/mm)	%age Error
		Loading	Unloading	Mean			
1							
2							
3							
4							
5							
6							

Name: _____

Reg. # _____

Date:

Report:

The laboratory report should contain the following:

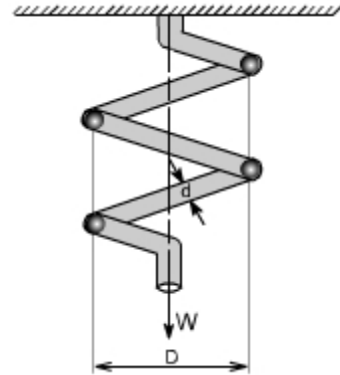
1. Plot of curve between Load **P** and Extension Δ as shown in figure (b). Calculate the slope of the graph.
2. Derivation of the formula.
3. Hand calculations showing all results under procedure above.
4. A discussion / comments of factors affecting the results of the experiment.
5. Practical Applications



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Derivation of The Formula: A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released. **or** Springs are energy absorbing units whose function is to store energy and to restore it slowly or rapidly depending on the particular application.

In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W .



Let

W = axial load

D = mean coil diameter

d = diameter of spring wire

N = number of active coils

l = length of spring wire = πDN ---- (i)

G = modulus of rigidity

Δ = deflection of spring

Φ = Angle of twist

In 1879, Alberto Castigliano, an Italian railroad engineer, published a book in which he outlined a method for determining the displacement / deflection & slope at a point in a body. This method which referred to Castigliano's Theorem applied to the bodies, have constant temperature & material with linear elastic behaviour. It states that "The derivative of the strain energy with respect to the applied load gives the deformation corresponding to that load**".

For a helical spring, the partial derivative of the strain energy w.r.t. the applied load gives the deflection in the spring i.e. $\partial U / \partial W = \text{deflection}$.

Consider a helical compression spring made up of a circular wire and subjected to an axial load W as shown in the figure above.

Strain Energy is given by:

$$U = \frac{1}{2} T * \Phi \quad \text{----- (ii)}$$

Where as,

$$T = \frac{1}{2} W * D \quad \text{----- (iii)}$$

$$\Phi = Tl / JG \quad \text{----- (iv) (From Torsion formula)}$$

putting the values from eqs. # (i), (iii) & (iv) in eq. # (ii) and simplifying, we get;

$$U = 4 W^2 D^3 N / d^4 G \quad \text{----- (v)}$$

Now applying the Castigliano's theorem by taking the partial derivative of the strain energy with respect to the applied load

$$\partial U / \partial W = \Delta = 8 W D^3 N / d^4 G \quad \text{----- (v)}$$

$$\rightarrow W / \Delta = d^4 G / 8 D^3 N$$

$$\rightarrow \text{Stiffness} = K = d^4 G / 8 D^3 N$$

{** load mean a force, a bending or a twisting effect / moment, where as the deformation is linear displacement, deflection or an angle of twist.}